2022-2023 Junior Mathematical Olympiad

SOLUTIONS: Final Round Examination (Grades 5 and 6)

1. **Soln**: **(D)** $2^2 \times 2^{2020} \times 2 = 2^2 \times 2^{2020} \times 2^1 = 2^{2+2020+1} = 2^{2023}$

2. **Soln**: **(C)** Since 60 minutes are in an hour, the number of beats in an hour is $70 \times 60 = 4,200$ times per hour.

3. **Soln**: **(E)** If the sum of the digits is a multiple of 3 then the number is a multiple of 3. The sum of the digits are respectively $6, 9, 12, 15, 18$. All are multiples of 3. The answer is 5.

4. **Soln**: **(A)** $\dfrac{66}{77} = \dfrac{6\,(11)}{7\,(11)} = \dfrac{6}{7}$. Similarly the other fractions are $\dfrac{5}{6}, \dfrac{4}{5}$ and $\dfrac{3}{4}$. The greatest is $\dfrac{7}{8}$.

5. **Soln**: **(A)** The sum of the 4 sides of the square is $4 \times 10 = 40$ cm. The sum of the 4 sides of the rectangle is $3 + 3 + 10 + 10 = 26$ cm. The difference in length is 40 cm $-$ 26 cm $=$ 14 cm

6. **Soln**: **(D)** Because Kassie and Zoe were born in the same month, they were both born in March. Because Julie and Zoe were born on the same day of a month, they were both born on the 20th. This means that Helen (the only girl left) was born May 17th

7. **Soln**: **(B)** 2/3 of the chocolate bar was used to feed each of her x children. Therefore $\frac{1}{12} \cdot x = \frac{2}{3}$ and $x = \frac{2}{3} \times 12 = 8$.

8. **Soln**: **(E)** The six marked angles are the interior angles of 2 triangles. The interior angles of 1 triangle add up to$180°$. So the marked angles add up to $2 \times 180° = 360°$.

9. **Soln**: **(D)** From the information given 4 pecks $=$ 1 bucket and 9 buckets $=$ 1 barrel. Since $4 \times 9 = 36$ pecks $=$ 9 buckets, 36 pecks $=$ 1 barrel. Peter already picked one peck an so he must pick an additional 35 pecks of peppers.

10. **Soln**: **(E)** Let the rectangle be a square (it does not matter) of sides 1 m $=$ 100 cm. The triangle cut off will have an area of $\frac{1}{2}\,(50)\,(50) = 1,250$ cm^2 (one-eighth of the rectangle)

11. **Soln**: **(E)** Since Diana is 3 years old and her mother is 28 years older than her. Mother is presently $3 + 28 = 31$ years. In x years, Diana will be $3 + x$ years old and her mother will be $31 + x$ years old. If we solve $3\,(3 + x) = 31 + x$, we get $9 + 3x = 31 + x$ or $2x = 22$ or $x = 11$.

12. **Soln**: **(A)** Let the length of one side of the square be 3 units so that the perimeter is $4 \times 3 = 12$. The perimeter of the octagon is $4+3+4+1+3+1+3+1 = 20$ The ratio is $12 : 20$ or $3 : 5$.

13. **Soln**: **(D)** Since the remaining parts from using four plates can be used to make one more plate, exactly 3 plates are used to make 4 medals. That is, one medal is made from 3/4 plate of gold. Now $16 = 3 \times 5 + 1$. Fifteen plates will make exactly $5 \times 4 = 20$ medals and the extra plate (the 16th) will make 1 medal. The total is $20 + 1 = 21$.

14. **Soln**: **(D)** Working in minutes, the match lasted $11 \times 60 + 5 = 665$ and the 5th set lasted $8 \times 60 + 11 = 491$. The fraction is

$$\frac{491}{665} \approx \frac{490}{665} = \frac{98}{133} = \frac{7 \times 7 \times 2}{7 \times 19} = \frac{14}{19} \approx \frac{15}{20} = \frac{3}{4}.$$

15. **Soln**: **(B)** $\dfrac{83}{4} = 20.75$ and so the winner must get 21 or more votes. If person 1 receives 21 votes, 62 votes must be shared between 3 persons and since $20+20+20 = 60$, at least one other person would get 21 or more votes and so the winner must get more than 21 votes. If person 1 gets 22 votes, the other three persons could get $20, 20, 21$ (for example) and person 1 wins.

16. **Soln**: **(B)** If the three digit number is divisible by 25 it must end in $00, 25, 50$ or 75.

    Case 1: Ending in 00 : The first digit can be $3, 5$ or 7 (3 in total)

    Case 2: Ending in 25 : Not possible (0 in total)

    Case 3: Ending in 50 : The first digit can be $3, 5$ or 7 (3 in total)

    Case 4: Ending in 75 : The first digit can be $3, 5$ or 7 (3 in total)

    The number of possibilities is $3 + 3 + 3 = 9$.

17. **Soln**: **(C)** The area of pentagon $ABCED$ is

$$\text{Area}\Delta ABD + \text{Area}\Delta ABC - \text{Area}\Delta ABE = 15 + 12 - 4 = 23.$$

18. **Soln**: **(A)** Only when 5 balls are in the bag can we guarantee that she took at least one ball of each color. This would require her to draw at least $14 + 8 + 6 - 5 = 23$ balls.

19. **Soln**: **(A)** Let D be the distance one way. The time in hours taken to go up and down the hill are respectively $\dfrac{D}{12}$ and $\dfrac{D}{20}$. So $\dfrac{D}{12} = \dfrac{D}{20} + \dfrac{16}{60}$. Multiplying both sides by 60, we get $5D = 3D + 16$. This gives $D = 8$ km. The time in minutes the cyclist takes to go down the hill is $\dfrac{D}{20} \times 60 = \dfrac{8}{20} \times 60 = 24.$ minutes.

20. **Soln**: **(C)** The length of time between sunrise and sunset is 9 : 25 P.M − 4 : 53 A.M. This is $7:07+9:25=16:32$. One half of this is $8:16$. **Local noon** is therefore 4 : 53 A.M. + 8 : 16 = 1 : 09 P.M

21. **Soln**: **(D)** Because three Sundays are on even days in this month, there are 5 Sundays in this month and the order is
$$E, O, E, O, E$$
where E represents even and O represents odd. If the first Sunday is on the 2nd then the last Sunday is on the 30th. If the first Sunday is on the 4nd then the last Sunday is on the 32nd. Since no month has 32 days, the first Sunday must be on the 2nd. The 6th of the month will therefore be a Thursday and so is the 20th.

22. **Soln**: **(A)** After day 1, the fraction destroyed is $\frac{1}{2}$. After day 2, the fraction destroyed is $\frac{1}{2}+\frac{1}{3}\cdot\frac{1}{2}=\frac{2}{3}$ The fraction remaining is $\frac{1}{3}$. After day 3, the fraction destroyed is $\frac{2}{3}+\frac{1}{4}\cdot\frac{1}{3}=\frac{3}{4}$ and the fraction remaining is $\frac{1}{4}$. After day 4, the fraction destroyed is $\frac{3}{4}+\frac{1}{5}\cdot\frac{1}{4}=\frac{4}{5}$ and the fraction remaining is $\frac{1}{5}$.

23. **Soln**: **(B)** Let the three digit number be abc. The sum of the digits is the number $a+b+c$ and
$$1 \leq a+b+c \leq 27$$
Note that $a+b+c=1$ for the number 100 and is equal to 27 for the number 999.

    For numbers in the range 1 to 27, the greatest value of the sum of the digits is 10 (corresponding to when $a+b+c=19$)

24. **Soln**: **(C)** Step 1 has 8 groups of 4 players. In each group with players (a, b, c, d). The pairs are
$$(a, b), (a, c), (a, d), (b, c), (b, d) \text{ and } (c, d)$$
The number of games in each group is 6. Step 1 has $6\times 8=48$ games. Step 2 has $6\times 4=24$ games. Step 3 has $6\times 2=12$ games. Step 4 has $6\times 1=6$. Step 5 has 1 game (the final). The total number of games played is $48+24+12+6+1=91$.

25. **Soln**: **(A)** Let t, s and c be the weights of the triangular, square and circular object. We have $P<Q<R$ and so
$$2t+s<2c+s<t+2s$$
From this, $t<c$ and $t<s$. The weight of S is
$$t+c+s<c+c+s=2c+s$$
So $S<Q$. Also, the weight of S is
$$t+c+s>t+t+s=2t+s$$
So $S>P$. Therefore $P<S<Q$.