2022-2023 Junior Mathematical Olympiad

Round One Solutions (Grades 5 and 6)
10:00am

For each question, determine the letter corresponding to the correct or best response; along with the question number, indicate this letter by shading it

1. **Soln**: **(E)** Since $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16}$, the sum is $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2$

2. **Soln**: **(C)** The next 4 terms of the sequence are $39, 28, 17, 6$.

3. **Soln**: **(C)** Because $4 \times 15 = 60$, John walks at rate $\dfrac{52}{4} = 13$ steps per 15 seconds. Since John walks 52 steps in 60 seconds and 13 steps in 15 seconds, he walks $52 + 13 = 65$ steps in $60 + 15 = 75$ seconds.

4. **Soln**: **(E)** The combined amount is $\$150 + \$200 + \$250 + \$400 = \$1000$. They each end up with $\$\frac{1000}{4} = \$250$

5. **Soln**: **(E)** The number of 10 cents coins is $\dfrac{100}{10} = 10$ and the number of 25 cents coins is $\dfrac{100}{25} = 4$. The total number of coins is therefore $10 + 4 = 14$ and the number of 10 cents coins to the total number of coins in the sack is 10 to 14 or 5 to 7.

6. **Soln**: **(B)** The area of the walkway is the area of the outer rectangle minus the area of the inner rectangle. This is $22 \times 10 - 20 \times 8 = 60$

7. **Soln**: **(E)** 1 cup of lemon juice requires 2 cups of sugar and 8 cups of water. 3 cups of lemon juice requires $3 \times 8 = 24$ cups of water.

8. **Soln**: **(D)** The size of the garden is $6 \times 8 = 48$ square metres. The total number of tomato plants is therefore $48 \times 4 = 192$. Since each plant on average yields 10 tomatoes, the total expected yield is 1920 tomatoes.

9. **Soln**: **(A)** When $n = 4, 1 + 3n\,(n-1) = 1 + 3 \times 4\,(4-1) = 1 + 12 \times 3 = 37$

10. **Soln**: **(C)** The fraction is $\frac{1}{5} \times \frac{3}{4} = 3/20$

11. **Soln**: **(E)** The factors of 42 are $1, 2, 3, 6, 7, 14, 21, 42$ and the sum is 96

12. **Soln**: **(D)** Since triangle PQR is isosceles, $\angle PQR = \angle PRQ = x^\circ$ and so $70+x+x = 180$. Since $2x = 110$, $x = 55$. Because $QRST$ is a rectangle, $y = 90$ and $x + y = 55 + 90 = 145$.

13. **Soln**: **(C)** With exactly one 7, there are

$$17, 27, 37, 47, 57, 67, 70, 71, 72, 73, 74, 75, 76, 78, 79, 87, 97.$$

Also 77 has exactly two sevens. The total number is 18.

14. **Soln**: **(C)** Since \$7.50 buys 250 grams of salt, \$1 buys $\frac{250}{7.50}$ grams of salt and \$1.80 buys $1.80 \times \frac{250}{7.50} = 60$ grams of salt.

15. **Soln**: **(B)** If the length of one side of square is x m, then the area of the rectangle is $3x^2$ and the perimeter is $8x = 56$. This gives $x = 7$. The area of the rectangle is $3x^2 = 3\times 7^2 = 147$

16. **Soln**: **(B)** Since the first Wednesday can be on the 1st, 2nd, 3rd, 4th, 5th, 6th or 7th, adding 14, the third Wednesday must be on the 15th, 16th, 17th, 18th, 19th, 20th or 21st.

17. **Soln**: **(C)** Figure 1 has $2 + 5 \times 1 = 7$ squares. Figure 2 has $2 + 5 \times 2 = 12$ squares. Figure 3 has $2 + 5 \times 3 = 17$ squares. Figure ℓ has $2 + 5\ell = 2022$ squares. From this $5\ell = 2020$. Dividing by 5 gives $\ell = 404$.

18. **Soln**: **(C)** Without the 40 minutes break, the total time for the journey is $(11 - 7) \times 60 - 40 = 200$ minutes. This is $\frac{200}{60} = \frac{10}{3}$ hours. The average speed is

$$\text{Average speed } = \frac{\text{Total distance}}{\text{Time taken}} = \frac{300 \text{ km}}{10/3 \text{ hours}} = 300 \times \frac{3}{10} = 90 \text{ km/h}$$

19. **Soln**: **(C)** After dividing 128 by 26, we get 4 with a remainder of 24 and so $128 = 26\times 4+ 24$. The 128th letter therefore correspond to the 24th letter in the 5th cycle. The 24th letter is X.

20. **Soln**: **(B)** Each package contains 2 more envelopes than cards. Since Kimmone has 7 cards, she must make 4 package purchases to net $4 \times 2 = 8$ envelopes which is more than 7. Note that 3 packages will result in Kimmone having $7 + 3 \times 8 = 31$ cards and 30 envelopes and 4 packages will result in Kimmone having $7 + 4 \times 8 = 39$ cards and 40 envelopes.

21. **Soln**: **(C)** The greatest amount is when the number of \$5 coins is most (2021 of them). The least amount is when the number of \$5 coins is least (1 of them). The difference is

$$(5 \times 2021 + 1 \times 1) - (5 \times 1 + 1 \times 2021) = 10\,106 - 2026 = 8080$$

22. **Soln**: (**E**) The possible dimensions of the rectangle are

$$24 \times 1, 12 \times 2, 8 \times 3 \text{ and } 6 \times 4$$

The corresponding perimeters are

$$2\,(24+1) = 50, 2\,(12+2) = 28, 2\,(8+3) = 22, 2\,(6+4) = 20.$$

23. **Soln**: **(A)** Based on the conditions: Dorian sat to the extreme right, Alexis sat directly to the right of Ellie and Bobbi sat somewhere to the left of Alexis, we have the following possible seating arrangements

B		E	A	D
	B	E	A	D
B	E	A		D

The condition that at least one person sat between Canute and Bobbi means that the only possible seating arrangement is BEACD

24. **Soln**: **(C)** The number of choices for the first digit is 1 (it must be 2). The number of choices for the second digit is 1 (it must be 3). The number of choices for the third digit is 5 $(4, 5, 6, 7, 8)$.

Case: Third digit is a 4 : The last digit is either $5, 6, 7, 8, 9$

Case: Third digit is a 5 : The last digit is either $6, 7, 8, 9$

Case: Third digit is a 6 : The last digit is either $7, 8, 9$

Case: Third digit is a 7 : The last digit is either $8, 9$

Case: Third digit is a 8 : The last digit is 9

The total is $5 + 4 + 3 + 2 + 1 = 15$

25. **Soln**: (**C**) Suppose the value of P is less than 9, then the maximum value of the sum $QR + PPP + PPP$ is $99 + 888 + 888 = 1875$. Since this value is less than 2022, the value of P must be 9 and so $QR + 999 + 999 = 2022$. From this, $QR = 2022 - 999 - 999 = 24$. So $P = 9, Q = 2, R = 4$ and $P + Q + R = 15$