2020 Senior Mathematical Olympiad
Test for Grades 9, 10, and 11

NAME _____

GRADE _____

SCHOOL _____

STUDENT PHONE _____

EXAMINATION QUESTIONS

1) What time was it 2019 minutes after midnight on January 1, 2019?

   a) January 1 at 9:39 pm   b) January 1 at 11:59 pm   c) January 2 at 3:19 am

   d) January 2 at 9:39 am   e) January 2 at 6:01 pm

2) Which of the following numbers is not a perfect square?

   a) 1^{2016} b) 2^{2017} c) 3^{2018} d) 4^{2019} e) 5^{2020}

3) In the figure below, four large squares of equal area have been subdivided into congruent rectangles or triangles. What percent of the combined area of the four squares is shaded?
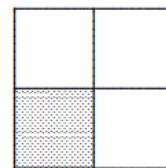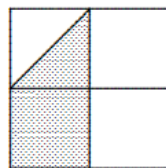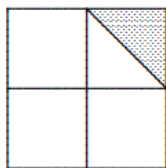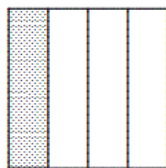
   a) $12\frac{1}{2}$ b) 20   c) 25   d) $33\frac{1}{3}$ e) $37\frac{1}{2}$

4) Alicia always lies on Mondays, Wednesdays, and Thursdays, and always tells the truth on the other days of the week. Brett always lies on Thursdays, Fridays, and Saturdays, and always tells the truth on the other days. One day, Alicia said, "Today is Monday", and Brett said, "Yes, today is Monday". Which day of the week was it?

   a) Friday   b) Sunday   c) Monday   d) Wednesday   e) Thursday

5) How many ordered pairs (m, n), where m and n are positive integers, satisfy the equation $3m + n = 100$?

      a) 33   b) 35   c) 100   d) 101   e) 97

6) The least common multiple of a and b is 12, and the least common multiple of b and c is 15. What is the least possible value of the least common multiple of a and c?

      a) 20   b) 30   c) 60   d) 120   e) 180

7) In a jar of red, green, and blue marbles, all but 6 are red marbles, all but 8 are green, and all but 4 are blue. How many marbles are in the jar?

      a) 6   b) 8   c) 9   d) 10   e) 12

8) Justin has a rectangular sheet of paper measuring 7×6. He wants to cut the paper into pieces so that each piece is a square. (The side length of each square should be a positive integer.) What is the least number of squares Justin could obtain?

      a) 4   b) 5   c) 7   d) 8   e) 42

9) What is the ones digit of the product $(5^2 + 1)(5^3 + 1)(5^{23} + 1)$?

      a) 0   b) 1   c) 2   d) 5   e) 6

10) A class collects $500 to buy flowers for a classmate who is in the hospital. Roses cost $30 each and carnations cost $20 each. No other flowers will be used. How many different bouquets could be purchased for exactly $500?

      a) 1   b) 7   c) 9   d) 16   e) 17

11) A chocolate drink is 6% pure chocolate, by volume. If 10 litres of pure milk are added to 50 litres of this drink, what percent of the new mixture will be pure chocolate?

      a) 5   b) 16   c) 10   d) 3   e) 26

12) Ximena wrote the numbers from 1 to 30 on a sheet of paper. Emilio copied her numbers, except that he wrote a 1 in each place Ximena had written a 2. Ximena summed the numbers she wrote and Emilio summed the numbers he wrote. How much larger was Ximena's sum than Emilio's?

      a) 13   b) 26   c) 102   d) 103   e) 110

13) What is the area of the shaded region in the 5×5 grid on the right?

      a) 6   b) 4   c) 8

      d) 12   e) 10
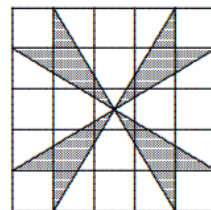
14) A sequence a_1, a_2, a_3, ... has the property that, for each positive integer n, the mean of its first n terms is n. What is the 2019th term of this sequence?

      a) 2019   b) 4037   c) 4038   d) 4,074,342   e) 4,076,361

15) Dwight decided to fence in his rectangular garden. He bought 20 fence posts, placed one on each of the four corners, and spaced out the rest evenly along the edges of the garden. There were exactly 4 metres between neighbouring posts. The longer side of his garden, including the corners, has twice as many posts as the shorter side, including the corners. What is the area of Dwight's garden?

a) $256\,\mathrm{m}^2$ b) $512\,\mathrm{m}^2$ c) $384\,\mathrm{m}^2$ d) $448\,\mathrm{m}^2$ e) $336\,\mathrm{m}^2$

16) At Urbana Hospital one year, 1000 of the babies born were members of a set of twins, triplets, or quadruplets. There were four times as many sets of triplets as sets of quadruplets, and there were three times as many sets of twins as sets of triplets. How many of these 1000 babies were in sets of quadruplets?

a) 25   b) 40   c) 64   d) 100   e) 160

17) Five friends sat in a row of five seats in a movie theatre. They noticed that their seat numbers went from 1 on the left to 5 on the right. At intermission, Ada bought some popcorn. When she returned, she found that Bea had moved 2 seats to the right, Ceci had moved 1 seat to the left, and Dee and Edie had switched places, leaving and end seat for Ada. In which seat was Ada sitting before she got up?

a) 1   b) 2   c) 3   d) 4   e) 5

18) The figure on the right shows four large circles and one small circle. Each large circle has the same radius and is tangent to two other large circles and the small circle. If the radius of the small circle is 1, what is the radius of a large circle?
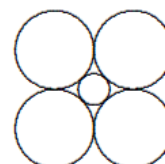

a) $\sqrt{2}$ b) $1+\sqrt{2}$ c) $\sqrt{6}$ d) 3   e) $2+\sqrt{2}$

19) Suppose a, b, and c are nonzero real numbers such that $a+b+c=0$. What are all the possible values for

$$\frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{abc}{|abc|} \ ?$$

a) 0   b) 1 and -1 c) 2 and -2

d) 0, 2, and -2 e) 0, 1, and -1

20) Sarah wants to write a positive integer in each small square in the figure on the right. After she completes the bottom row, she wants each number she places in a higher square to be the sum of the two numbers in the squares just below it. What is the largest number of odd numbers Sarah could place in a completed table?
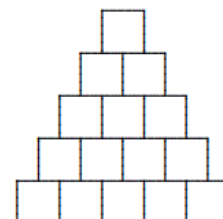
a) 5   b) 7   c) 8   d) 10   e) 11

21) Suppose a, b, and c are nonzero real numbers. If $xy = a$, $xz = b$, and $yz = c$, which expression below is equal to $x^2 + y^2 + z^2$?

a) $\dfrac{ab + ac + bc}{abc}$

b) $\dfrac{a^2 + b^2 + c^2}{abc}$

c) $\dfrac{(a + b + c)^2}{abc}$

d) $\dfrac{(ab + ac + bc)^2}{abc}$

e) $\dfrac{(ab)^2 + (ac)^2 + (bc)^2}{abc}$

22) In a recent basketball game, Shenille attempted only three-point shots and two-point shots. She was successful on 20% of her three-point shots and 30% of her two-point shots. Shenille attempted 30 shots. How many points did she score?

a) 12   b) 18   c) 24   d) 30   e) 36

23) Squares of size $1\,\text{cm} \times 1\,\text{cm}$ are cut from the corners of the $5\,\text{cm} \times 5\,\text{cm}$ square shown on the right. What is the area of the largest square that can be fit into the remaining space?

a) $15\frac{1}{2}\,\text{cm}^2$ b) $9\,\text{cm}^2$ c) $17\,\text{cm}^2$ d) $12\frac{1}{2}\,\text{cm}^2$ e) $15\,\text{cm}^2$
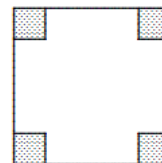
24) In the sequence 2001, 2002, 2003, $\ldots$, each term after the third one is found by subtracting the previous term from the sum of the two terms that precede that term. For example, the fourth term is $2001 + 2002 - 2003 = 2000$. What is the 2004th term of this sequence?

a) -2004 b) -2 c) 0   d) 4003   e) 6007

25) The square on the right is a multiplicative magic square. This means that the product of the numbers in each row, column, and diagonal is the same. If all the entries in the table are positive integers, what is the sum of the possible values of g?

50	b	c
d	e	f
g	h	2

a) 35   b) 10   c) 25   d) 136   e) 62

End of Questions

You may mail this completed question paper to:

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You may also deliver your entry by hand or by courier directly to the Department of Mathematics at the UWI, Mona Campus. Each entry should be accompanied by a $500 Entrance Fee. In all cases, an entry must be received by December 9, 2019, to be guaranteed consideration.