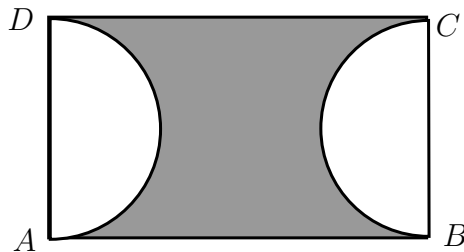


2022-2023 Senior Mathematical Olympiad

Final Round Examination (Grades 7 and 8)

Provide complete solutions to all 7 questions (2 hours)

1. Find all pairs of positive integers (x, y) where the ratios $x : 4$ and $9 : y$ are equal?
2. A sum of money is being divided among Altiman, Britannie and Cecil. First, Altiman receives \$100 plus one-third of what is left. Britannie then receives \$600 plus one-third of what remains. Finally Cecil receives \$4000 which is the remaining amount. What is the total amount of money that is being shared?
3. The diagram shows a rectangle $ABCD$ with $AD = 10$ cm and the area of the shaded area is 100 cm^2 .



What is the shortest distance between the two semicircles?

4. What is the value or values of the digit k that makes the five-digit number

$$275k2$$

divisible by 12?

5. An **altitude (whose length is called height)** of a triangle is the perpendicular drawn from one vertex of the triangle to the opposite side.

The sides of a triangle have lengths 30 cm, 40 cm and 50 cm. Calculate the length of the shortest altitude?

It may help to know that $30^2 + 40^2 = 50^2$.

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6. Consider the numbers

$$\begin{aligned}w &= 2^{129} \times 3^{81} \times 5^{131}, \\x &= 2^{127} \times 3^{81} \times 5^{131}, \\y &= 2^{126} \times 3^{82} \times 5^{131}, \\ \text{and } z &= 2^{125} \times 3^{82} \times 5^{132}\end{aligned}$$

Show the necessary work to demonstrate that x is smallest and w is largest.

7. Freddie has a number of square tiles, each measuring 1 cm by 1 cm. He tries to put these small square tiles together to form a larger square of side length n cm, but finds that he has 92 tiles left over. If he had increased the side length of the larger square to $(n + 2)$ cm, he would have been 100 tiles short of completing the larger square. How many tiles does Freddie have?