2022-2023 Senior Mathematical Olympiad

Round One Examination (Grade 7 and 8) Solutions

1. **Soln**: **(B)** $20 + 22 = 42$ and $20 - 22 = -2$. The answer is $\dfrac{42}{-2} = -21$

2. **Soln**: **(A)** We have $(6\blacksquare 3) + 4 - 1 = 5$ or $(6\blacksquare 3) = 2$. Of the symbols given, $\blacksquare$ represents the $\div$ symbol.

3. **Soln**: **(E)** $360 = 3 \cdot 120 = 3 \cdot 4 \cdot 30 = 3 \cdot 4 \cdot 5 \cdot 6$

4. **Soln**: **(E)** Alex and Barry together are 11 years old. This makes Carl age $15 - 11 = 4$. Barry and Carl together are 12 years old. This makes Alex age $15 - 12 = 3$. Barry's age is therefore $15 - (4 + 3) = 8$ years old.

5. **Soln**: **(A)** $2022 = 2 \times 1011 \times 1$. This result in the largest value of $J + M + O = 2 + 1011 + 1 = 1014$

6. **Soln**: **(C)** Since angles on a straight line adds to $180°$, the base angles of the triangle are $65°$ and $55°$. The sum of the angles in a triangle is $180°$ and so $x = 180 - (65 + 55) = 60$

7. **Soln**: **(B)** The number is smallest when the leading digits/cards are arranged smallest to largest from left to right. In order, they are

$$107, 31, 4, 59, 8$$

8. **Soln**: **(C)** Fogo lands on the numbers

$$3, 6, 7, 8, 9, 12, 15, 16, 17, 18, \ldots$$

It is now evident that he lands on all the positive integers that are divisible by 3. Of the numbers $84 = 3 \cdot 28$ and so he lands on 84. Note that he land on

$$\ldots 81, 84, 87, 88, 89, 90, \ldots$$

9. **Soln**: **(B)** The minimum number of packages is bought when Tom buys 3 packages of 25 marbles and 2 packages of 10 marbles. This purchase results in exactly 95 marbles. Any other purchase will result in more packages being bought.

10. **Soln**: (**D**) Let $QY = x$. The area below PQ is the area of the triangle plus the area of 1 square. This must be equal to 5 squares. So

$$1 + \frac{1}{2} \cdot 5\,(1 + x) = 5$$

This gives $2 + 5\,(1 + x) = 10$ and so $1 + x = \frac{8}{5}$ or $x = \frac{3}{5}$. Since $QY = x = \frac{3}{5}$, $XQ = 1 - x = \frac{2}{5}$ and the ratio $\dfrac{XQ}{QY} = \dfrac{2}{3}$.

11. **Soln**: (**D**) Let the dimension of the smaller rectangle be $x \times y$ where $x > y$. From the diagram, $4y = 3x$ or $y = \frac{3}{4}x$

$$\frac{AB}{BC} = \frac{4y}{x + y} = \frac{3x}{x + \frac{3}{4}x} = \frac{12x}{4x + 3x} = \frac{12}{7}$$

12. **Soln**: (**C**) At 2 o'clock, the minute hand is at 12 and the hour hand is at 2. Each unit difference on the clock represents $\frac{1}{12} \times 360 = 30$ and so the angle represented is $2 \times \frac{1}{12} \times 360 = 60$.

13. **Soln**: (**A**) There is only one choice for the first digit (namely 1). There are 5 choices for the second digit (namely $1, 3, 5, 7, 9$) and there are 5 choices for the third digit (namely $1, 3, 5, 7, 9$). The total is $1 \times 5 \times 5 = 25$

14. **Soln**: (**B**) Dividing the rectangle into $2 \times 4 \times 2 = 16$ congruent triangles. The shaded region represents 4 such triangles. The fraction is $\frac{4}{16} = 1/4$

15. **Soln**: (**D**) The original area is $50\text{ m} \times 10\text{ m} = 500\text{ m}^2$. The perimeter is $2\,(50 + 10) = 120$ m. Made into a square, each side will be of length $\frac{120}{4} = 30$ m and the resulting area is $30\text{ m} \times 30\text{ m} = 900\text{ m}^2$. The increase is 400 m^2.

16. **Soln**: (**E**) From the second sentence, Cloe has more than someone so she can't have the least. From the third sentence both Ali and Boo have more than someone so that eliminates them. And, from the fourth sentence, Dre has more than someone, so that leaves Ezra with the least.

17. **Soln**: (**D**) Let f be the fixed monthly fee, h be the hourly rate and t, the total connect time in December. We have

$$f + ht = 1248 \text{ and } f + h \cdot 2t = 1754$$

From this, $2f + 2ht = 2496$ and $f + 2ht = 1754$. Subtracting, $f = 2496 - 1754 = 742$.

18. **Soln**: (**C**) With the straight line DB, we construct two right angled triangles. The area of the quadrilateral is now

$$\frac{1}{2}(AB)(AD) + \frac{1}{2}(CD)(CB) = \frac{1}{2}(11)(3) + \frac{1}{2}(9)(7) = 48$$

19. **Soln**: (**B**) If she wrote down n numbers then Rhianna's sum is 34 and so

$$7 \times n - 22 = 34$$

or $7n = 56$. Dividing through by 7 gives $n = 8$

20. **Soln**: (**E**) The square $PQRS$ may be thought of as comprising of 4 congruent kites (we want the area of two) and 4 congruent triangles. The total area of the triangles is $4 \times \frac{1}{2} \times 4 \times 6 = 48$. The area of the 4 congruent kites is therefore $12^2 - 48 = 96$. The area of two of them is $\frac{1}{2} \times 96 = 48$

21. **Soln**: (**E**) Let the outcomes on the first and second coin be $(5, x)$ and $(6, y)$. The possible sums are

$$5 + 6, 5 + y, x + 6 \text{ and } x + y$$

$x = 7$ and $y = 5$ produces the sum of the outcomes as $11, 10, 13$ and 12. The product is $7 \times 5 = 35$

22. **Soln**: (**E**) Bob's money increases only when he puts in a 1\$ and his net increase is \$199. Over time, the amount Bob has must take the form $\$1 + \$199n$ where n is the number of times a \$1 is inserted. Of the answers, only 996 can be written in this form. In fact, $996 = 1 + 199 \times 5$.

23. **Soln**: (**C**) In any one hour period the time differential between the two clocks is $1 + 2 = 3$ minutes. Today the time differential is 1 hour or 60 minutes. This means that $\frac{60}{3} = 20$ hours elapsed. It now means that the true time on Clock 1 is $12:00$ minus 20 mins or $11:40$. Finally 20 hours before $11:40$ is $35:40 - 20:00 = 15:40$

24. **Soln**: (**D**) Let Tom's dad present age be x. His mother's age is $x - 1$. Next year, their ages are $x + 1$ and x. So

$$x(x + 1) > 1000$$

Since $31 \times 32 = 992$ and $32 \times 33 = 1056$, $x = 32$.

25. **Soln**: (**E**) The weight of non-water in the watermelon is $\frac{2}{100} \times 20 = \frac{2}{5}$ kg. Let the present weight of the watermelon be x kg.

The $\frac{2}{5}$ kg non-water component now represents $5\% = \frac{1}{20}$ of x. The value of x is therefore $\frac{2}{5} \times 20 = 8$ kg