

# 2023 Senior Mathematical Olympiad

## Qualifying Round Examination (Grades 9, 10 and 11)

NAME\_\_\_\_\_

GRADE\_\_\_\_\_

SCHOOL\_\_\_\_\_

STUDENT CONTACT NUMBER\_\_\_\_\_

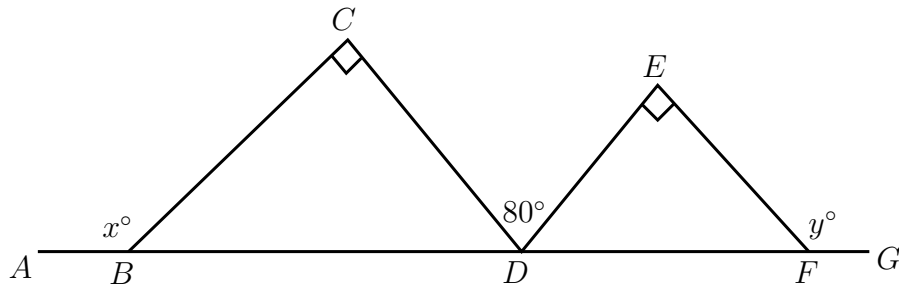
- EACH entry MUST be accompanied by a nominal entry fee of **J\$1000**
- Be sure to staple ALL pages (including this one) together
- All entries must reach the Mathematics Department, U.W.I by  
**Wednesday 13 December 2023**
- You may deliver by (a) Hand (b) Courier (c) Local Mail

- The Courier address is  
**Mathematics Department, UWI**  
**Mona**  
**Kingston 7**

- The Mailing address is  
**Senior Mathematical Olympiad**  
**P.O. Box 94**  
**Mona Post Office**  
**Kingston 7**

For each question, determine the letter corresponding to the correct or best response; along with the question number, indicate this letter by circling or shading it.

- The average of a list of 25 numbers is 21 and the average of a list of 5 numbers is 15. When the two lists are merged, what is the average of the resulting 30 numbers?  
 (A) 17    (B) 19    (C) 20    (D) 16    (E) 18
- Paddling upstream, it took Simone 2 hours and 30 minutes to get to her destination which is 4.5 km away. Simone's return trip took her  $\frac{1}{3}$  of the time it took her earlier. What is the average speed paddling downstream and upstream combined?  
 (A)  $1.25 \text{ kmh}^{-1}$     (B)  $3.96 \text{ kmh}^{-1}$     (C)  $1.8 \text{ kmh}^{-1}$     (D)  $1.95 \text{ kmh}^{-1}$     (E)  $2.7 \text{ kmh}^{-1}$
- Harry took a small pizza to the park. For his lunch, Harry ate  $\frac{1}{4}$  of the pizza. A cat came by and ate  $\frac{1}{3}$  of what Harry left behind. After that, a pigeon came and ate  $\frac{1}{3}$  of what the cat left behind. What fraction of the original pizza remained after the pigeon left?  
 (A)  $\frac{1}{12}$     (B)  $\frac{1}{6}$     (C)  $\frac{1}{4}$     (D)  $\frac{1}{3}$     (E)  $\frac{5}{12}$
- In the diagram shown,  $A, B, D, F$  and  $G$  are points on a horizontal line



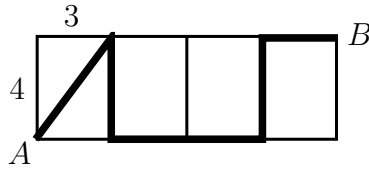
$\triangle BCD$  is right-angled at  $C$  and  $\triangle DEF$  is right-angled at  $E$ . The diagram also shows

$$\angle ABC = x^\circ, \angle CDE = 80^\circ \text{ and } \angle EFG = y^\circ.$$

What is the value of  $x + y$ ?

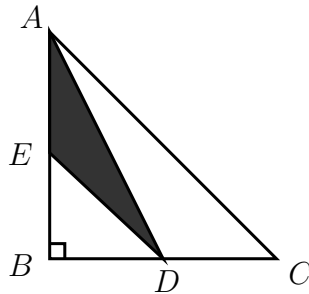
- (A) 250    (B) 260    (C) 270    (D) 280    (E) 290
- The letters  $a, b, c, d, e$  and  $f$  each represents one and only one of the numbers 1, 2, 3, 4, 5 and 6. The sum of  $a$  and  $b$  is 5 and the difference between  $c$  and  $d$  is 5. Of the list,  $e$  is the greatest in value. What is the value of  $e$ ?  
 (A) 4    (B) 6    (C) 2    (D) 3    (E) 5

6. The path shown in the diagram passes through 4 congruent rectangles of dimensions 3 and 4.



What is the length of the path from  $A$  to  $B$ ?

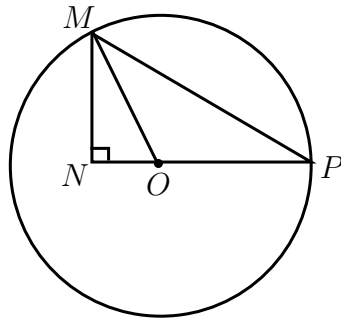
- (A) 22    (B) 21    (C) 19    (D) 20    (E) 23
7. Four points, labelled  $A, B, C$ , and  $D$  are placed on a line in the order  $A, B, C, D$ . The distance between  $D$  and  $A$  is 24. The distance between  $D$  and  $B$  is 3 times the distance between  $B$  and  $A$ . The point  $C$  is halfway between  $B$  and  $D$ . What is the distance between  $C$  and  $A$ ?
- (A) 12    (B) 8    (C) 16    (D) 9    (E) 15
8. The diagram below shows a right-angled isosceles triangle,  $\triangle ABC$ . The points  $D$  and  $E$  are midpoints of  $BC$  and  $AB$  respectively.



Given that  $AB = BC = 24$  cm, what is the area of  $\triangle AED$ ?

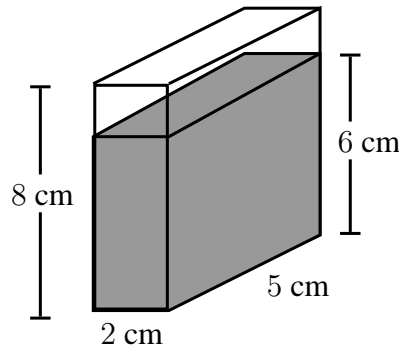
- (A)  $48 \text{ cm}^2$     (B)  $36 \text{ cm}^2$     (C)  $72 \text{ cm}^2$     (D)  $9 \text{ cm}^2$     (E)  $54 \text{ cm}^2$
9. Cylinder A, which is empty, has radius 6 cm and height 50 cm. Cylinder B, which is full of water, has radius 8 cm and height 50 cm. Water is poured from Cylinder B into Cylinder A until the height of water in both cylinders are the same. What is the height of the water in the cylinders after pouring?
- (A) 28.1 cm    (B) 25.0 cm    (C) 32.0 cm    (D) 44.4 cm    (E) 28.6 cm

10. The diagram below shows a circle with centre  $O$  and with radius 87. Points  $P$  and  $M$  lie on the circle and the point  $N$  is inside the circle so that  $PN$  passes through  $O$  and is perpendicular to  $MN$ .



Given that  $MN = 63$ , what is the area of the triangle  $PMN$ ?

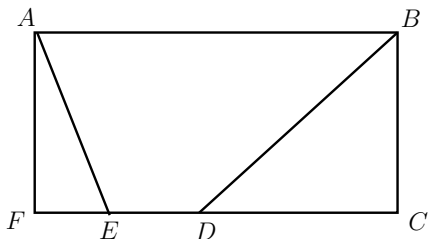
- (A) 3370.5    (B) 3496.5    (C) 4725.0    (D) 4630.5    (E) 4126.5
11. At 9 a.m., Pamella finished polishing  $\frac{1}{2}$  of her jewels. At 10 a.m., she finished polishing  $\frac{7}{8}$  of her jewels. If Pamella polishes her jewels at a constant rate, at what time did she finish polishing all her jewels?
- (A) 10:15 a.m.    (B) 11:20 a.m.    (C) 10:20 a.m.    (D) 10:30 a.m.    (E) 11:40 a.m.
12. The diagram below shows a closed rectangular prism with side dimensions, 2 cm, 5 cm and 8 cm. When standing on the 2 cm by 5 cm base, the depth of the water it contains is 6 cm.



When the prism is standing on the face with greatest area, what is the new depth of the water inside?

- (A) 0.75 cm    (B) 1 cm    (C) 1.25 cm    (D) 1.5 cm    (E) 1.75 cm

13. The diagram shows a rectangle,  $ABCF$ , with  $AB = 30$  cm and  $AF = 14$  cm.



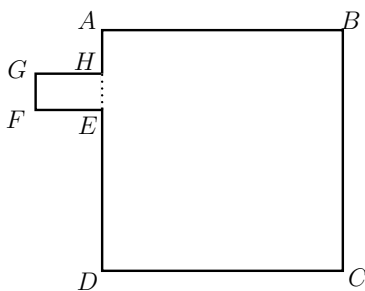
The points  $E$  and  $D$ , on  $FC$ , are such that  $FE = 5$  cm and the area of quadrilateral  $ABDE$  is  $266$  cm<sup>2</sup>. What is the length of  $DC$ ?

- (A) 16 cm    (B) 17 cm    (C) 18 cm    (D) 19 cm    (E) 20 cm
14.  $\overline{ab}$  and  $\overline{cd}$  is one pair of two-digit numbers. If the product  $\overline{ab} \times \overline{cd} = 630$ , how many such pairs of  $\overline{ab}$  and  $\overline{cd}$  are there?
- (A) 2    (B) 3    (C) 4    (D) 5    (E) 6
15. Let  $(m, n)$  be a pair of positive integers that satisfy the equation

$$\frac{m}{7} + \frac{2}{n} = 1.$$

How many such pairs  $(m, n)$  are there?

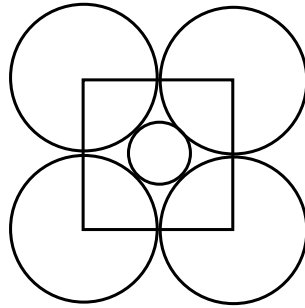
- (A) 0    (B) 1    (C) 2    (D) 4    (E) 5
16. The 8-sided polygon  $ABCDEFGH$  shown, has integer side lengths. As shown in the diagram, it can be divided into a rectangle and a square where the area of the square is greater than the area of the rectangle.



The product of the two areas is equal to 98. Of the following, which could be the perimeter of the polygon  $ABCDEFGH$ ?

- (A) 51    (B) 32    (C) 44    (D) 34    (E) 33

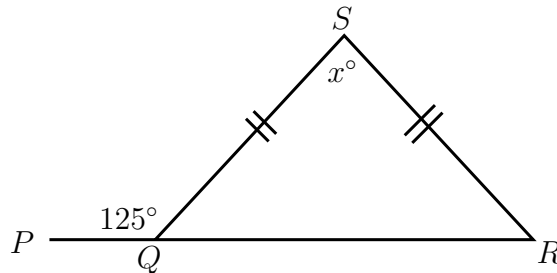
17. The diagram below shows 4 congruent circles, each of radius 5. They are arranged in such a way that their centres are vertices of a square.



A smaller circle is of radius  $r$  and touches all four larger circles as shown.

Which of the following is closest to the true value of  $r$ ?

- (A) 1.9    (B) 2.0    (C) 2.1    (D) 2.2    (E) 2.3
18. A square floor has a  $4 \times 4$  dimension and is to be covered with 16 square tiles. The tiles are coloured red, black, green, and yellow. Each row must contain one tile of each colour and no two tiles of the same colour should share a side or share a corner. In how many ways can the floor be tiled?
- (A) 256    (B) 24    (C) 120    (D) 576    (E) 64
19. In the diagram shown,  $PQR$  is a straight line segment,  $\angle PQS = 125^\circ$ ,  $\angle QSR = x^\circ$ , and  $SQ = SR$ .



What is the value of  $x$ ?

- (A) 60    (B) 70    (C) 80    (D) 110    (E) 125

20. A *Jayemo sequence* is a sequence of numbers in which each number after the second is the **non-negative difference (positive or zero)** between the two previous numbers. For example, if the first two numbers in a Jayemo sequence are 15 and 12, then the third number in the sequence is  $15 - 12 = 3$ , the fourth number is  $12 - 3 = 9$ , the fifth number is  $9 - 3 = 6$ . So the resulting sequence is 15, 12, 3, 9, 6, . . . . If a Jayemo sequence begins 10, 8, what is the sum of the first 90 numbers in the sequence?

- (A) 96      (B) 102      (C) 120      (D) 124      (E) 144

21. Gumbo gums are sold in packages of 5 gums and in packages of 8 gums. Helena can purchase exactly 18 gums by buying two 5-packs and one 8-pack, but she cannot purchase exactly 12 gums with any combination of the two packages. For how many values in the following list of possible values of  $n$ ,

$$n = 24, 25, 26, 27, 28, 29$$

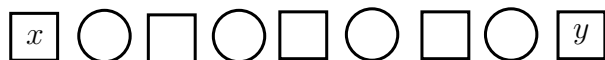
can she buy exactly  $n$  gums?

- (A) 5      (B) 3      (C) 2      (D) 4      (E) 6

22. Della writes down a list of four integers. From this, there are four subsets of three integers. She then calculates the average of each of the four subsets of three integers. These averages are 32, 39, 40, 44. What is the largest of the four integers?

- (A) 56      (B) 57      (C) 58      (D) 59      (E) 60

23. The integers 1, 2, 4, 5, 6, 9, 10, 11, 13 are to be placed in circles and squares (nine in total) with the condition that each integer is to be used exactly once and the number placed in a circle should be equal to the sum of the numbers placed in the adjacent (neighbouring) squares. Two of these integers  $x$  and  $y$  are placed in squares as shown.



Of all the possible configurations, what is the largest possible value of  $x + y$ ?

- (A) 4      (B) 8      (C) 12      (D) 16      (E) 20

24. The equation of a straight line is given as  $y = mx - 50$  where  $m$  is a positive integer. Let  $a$  be a given positive number. If the line passes through the point  $(a, 0)$  what is the sum of all the possible values of  $m$ ?
- (A) 73      (B) 83      (C) 93      (D) 103      (E) 113
25. The integers  $a$  and  $b$  satisfy the equation  $2a^2 + 8b = 26$ . Which of the following is a possible value of  $a - b$ ?
- (A)  $-8$       (B) 26      (C)  $-16$       (D) 22      (E) 30

**Please write your name here**\_\_\_\_\_